Course Number
Instructor
Date

Section 5.5 Multiple-Angle and Product-to-Sum Formulas

Objective: In this lesson you learned how to use multiple-angle formulas, power-reducing formulas, half-angle formulas, and product-to-sum formulas to rewrite and evaluate trigonometric functions.

I. Multiple-Angle Formulas (Pages 405–407)

What you should learn
How to use multiple-angle formulas to rewrite and evaluate trigonometric functions

The ___double-angle formulas___ are often used in trigonometry and calculus. These are listed below:

sin 2u = ___2 sin u cos u___

cos 2u = ___$\cos^2 u - \sin^2 u$___

= ___$2\cos^2 u - 1$___

= ___$1 - 2\sin^2 u$___

tan 2u = ___$(2 \tan u)/(1 - \tan^2 u)$___

To form other multiple-angle formulas, ___use 4θ and 2θ or 6θ and 3θ in place of 2θ and θ in the double-angle formulas or by using the double-angle formulas together with the appropriate trigonometric sum formulas___.

Example 1: Use multiple-angle formulas to express cos 3x in terms of cos x.

$4\cos^3 x - 3\cos x$

II. Power-Reducing Formulas (Page 407)

> ***What you should learn***
> How to use power-reducing formulas to rewrite and evaluate trigonometric functions

Power-reducing formulas can be used to write powers of trigonometric functions in a way that does not involve powers.

The power-reducing formulas are:

$\sin^2 u =$ $(1 - \cos 2u)/2$

$\cos^2 u =$ $(1 + \cos 2u)/2$

$\tan^2 u =$ $(1 - \cos 2u)/(1 + \cos 2u)$

III. Half-Angle Formulas (Pages 408–409)

> ***What you should learn***
> How to use half-angle formulas to rewrite and evaluate trigonometric functions

List the **half-angle formulas:**

$\sin \frac{u}{2} =$ $\pm \sqrt{(1 - \cos u)/2}$

$\cos \frac{u}{2} =$ $\pm \sqrt{(1 + \cos u)/2}$

$\tan \frac{u}{2} =$ $(1 - \cos u)/(\sin u)$ = $(\sin u)/(1 + \cos u)$

The signs of sin (u/2) and cos (u/2) depend on the quadrant in which u/2 lies.

Example 2: Find the exact value of tan 15°.

$2 - \sqrt{3}$

IV. Product-to-Sum Formulas (Pages 409–411)

> ***What you should learn***
> How to use product-to-sum and sum-to-product formulas to rewrite and evaluate trigonometric functions

The **product-to-sum formulas** can be used to rewrite products of trigonometric functions as a sum or difference.

The product-to-sum formulas are:

$\sin u \sin v =$ $1/2[\cos(u - v) - \cos(u + v)]$

$\cos u \cos v =$ $1/2[\cos(u - v) + \cos(u + v)]$

$\sin u \cos v =$ $1/2[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v =$ $1/2[\sin(u + v) - \sin(u - v)]$

Example 3: Write $\cos 3x \cos 2x$ as a sum or difference.

$1/2 \cos x + 1/2 \cos 5x$

The **sum-to-product formulas** can be used to rewrite a sum or difference of trigonometric functions as a product.

The sum-to-product formulas are:

$\sin u + \sin v =$ $2 \sin((u + v)/2) \cos((u - v)/2)$

$\sin u - \sin v =$ $2 \cos((u + v)/2) \sin((u - v)/2)$

$\cos u + \cos v =$ $2 \cos((u + v)/2) \cos((u - v)/2)$

$\cos u - \cos v =$ $-2 \sin((u + v)/2) \sin((u - v)/2)$

Example 4: Write $\cos 4x + \cos 2x$ as a sum or difference.

$2 \cos 3x \cos x$

Additional notes

Homework Assignment
Page(s)
Exercises