Course Number

Instructor

Date

## Section 10.5 Rotation of Conics

**Objective:** In this lesson you learned how to eliminate the *xy*-term in the equation of a conic and use the discriminant to identify a conic.

**Important Vocabulary** Define each term or concept.

**Invariant under rotation** A term or quantity in the equation of a conic that remains the same during a rotation of the coordinate axes through an angle $\theta$.

**Discriminant** The quantity $B^2 - 4AC$, of the general conic equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, which can be used to classify the type of conic.

***What you should learn***
How to rotate the coordinate axes to eliminate the *xy*-term in equations of conics

### I. Rotation (Pages 761–764)

The general equation of a conic whose axes are rotated so that they are not parallel to either the *x*-axis or the *y*-axis contains a(n) _____*xy*-term_____.

To eliminate this term, you can use a procedure called __rotation of axes_____, whose objective is to rotate the *x*- and *y*-axes until they are parallel to the axes of the conic.

The general second-degree equation

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ can be rewritten as

$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$ by rotating the coordinate axes through an angle $\theta$, where

$\cot 2\theta =$ _____$(A - C)/B$_____.

The coefficients of the new equation are obtained by making the substitutions $x =$ _____$x' \cos \theta - y' \sin \theta$_____ and $y =$ _____$x' \sin \theta + y' \cos \theta$_____.

## II. Invariants Under Rotation (Pages 765–766)

***What you should learn***
How to use the discriminant to classify conics

The rotation of the coordinate axes through an angle $\theta$ that transforms the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ into the form $A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$ has the following rotation invariants:

1) $F = F'$

2) $A + C = A' + C'$

3) $B^2 - 4AC = (B')^2 - 4A'C'$

The graph of the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is, except in degenerate cases, determined by its discriminant as follows:

1) Ellipse or circle if: $B^2 - 4AC < 0$

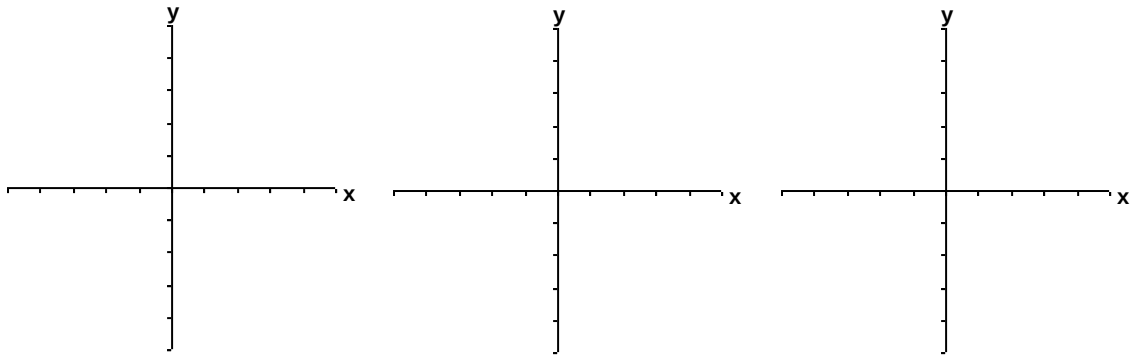
2) Parabola if: $B^2 - 4AC = 0$

3) Hyperbola if: $B^2 - 4AC > 0$

**Example 1:** Classify the graph of the following conic:

$2x^2 + 12xy + 18y^2 - 3y - 5 = 0$

Parabola




**Homework Assignment**

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