Chapter 12 Limits and an Introduction to Calculus

Course Number
Instructor
Date

Section 12.1 Introduction to Limits

Objective: In this lesson you learned how to estimate limits and use properties and operations of limits.

I. The Limit Concept and Definition of Limit (Pages 814–816)

What you should learn
How to use the definition of limit to estimate limits

Define **limit.**

If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, then the limit of $f(x)$ as x approaches c is L. This written as $\lim_{x \to c} f(x) = L$.

Describe how to estimate the limit $\lim_{x \to -2} \frac{x^2 + 4x + 4}{x + 2}$ numerically.

Let $f(x) = (x^2 + 4x + 4)/(x + 2)$. Then construct a table that shows values of $f(x)$ for two sets of x-values—one set that approaches -2 from the left and one that approaches -2 from the right. Use the table to look for a numerical trend in the value of $f(x)$ as x approaches -2. This is an estimate of the limit.

The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of the limit of $f(x)$ as x approaches c.

II. Limits That Fail to Exist (Pages 817–818)

What you should learn
How to determine whether limits of functions exist

The limit of $f(x)$ as $x \to c$ does not exist when any of the following conditions is true.

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side of c.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

Give an example of a limit that does not exist.

Answers will vary.

III. Properties of Limits and Direct Substitution
(Pages 819–821)

> ***What you should learn***
> How to use properties of limits and direct substitution to evaluate limits

Let b and c be real numbers and let n be a positive integer. Complete each of the following properties of limits.

1. $\lim_{x \to c} b =$ ___ b ___

2. $\lim_{x \to c} x =$ ___ c ___

3. $\lim_{x \to c} x^n =$ ___ c^n ___

4. $\lim_{x \to c} \sqrt[n]{x} =$ ___ $\sqrt[n]{c}$, for n even and $c > 0$ ___

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

Complete each of the following statements about operations with limits.

1. Scalar multiple: $\lim_{x \to c} [b\, f(x)] =$ ___ bL ___

2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] =$ ___ $L \pm K$ ___

3. Product: $\lim_{x \to c} [f(x) \cdot g(x)] =$ ___ LK ___

4. Quotient: $\lim_{x \to c} \dfrac{f(x)}{g(x)} =$ ___ L/K, provided $K \neq 0$ ___

5. Power: $\lim_{x \to c} [f(x)]^n =$ ___ L^n ___

Example 1: Find the limit: $\lim_{x \to 4} 3x^2$.

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If p is a polynomial function and c is a real number, then

$\lim_{x \to c} p(x) =$ ______$p(c)$______.

If r is a rational function given by $r(x) = p(x)/q(x)$, and c is a real number such that $q(c) \neq 0$, then

$\lim_{x \to c} r(x) =$ ______$r(c) = p(c)/q(c)$______.

Example 2: Find the limit: $\lim_{x \to 2} \frac{4 - x^2}{x}$.

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Additional notes

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Homework Assignment

Page(s)

Exercises