

## Section 12.2 Techniques for Evaluating Limits

**Objective:** In this lesson you learned how to find limits by direct substitution and by using the dividing out and rationalizing techniques.

Course Number

Instructor

Date

### I. Dividing Out Technique (Pages 825–826)

Describe what is meant by the term **indeterminate form**.

The fraction  $0/0$  that results when direct substitution produces  $0$  in both the numerator and the denominator. It has no meaning as a real number and is called an indeterminate form because it is not possible to determine the limit from the form alone.

***What you should learn***

How to use the dividing out technique to evaluate limits of functions

When you try to evaluate a limit of a rational function by direct substitution and encounter the indeterminate form  $0/0$ , you can conclude \_\_\_\_\_ that the numerator and denominator must have a common factor \_\_\_\_\_.

The validity of the dividing out technique stems from \_\_\_\_\_ the fact that when two functions agree at all but a single number  $c$ , they must have identical limit behavior at  $x = c$  \_\_\_\_\_.

**Example 1:** Find the limit:  $\lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x - 3}$ .  
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### II. Rationalizing Technique (Page 827)

Another way to find the limits of some functions is to first rationalize the numerator of the function. This is called the \_\_\_\_\_ rationalizing technique \_\_\_\_\_.

***What you should learn***

How to use the rationalizing technique to evaluate limits of functions

Recall that rationalizing the numerator means \_\_\_\_\_ multiplying the numerator and denominator by the conjugate of the numerator \_\_\_\_\_.

**III. Using Technology** (Page 828)

To find limits of nonalgebraic functions, \_\_\_\_\_ you often need to use more sophisticated analytic techniques such as a numerical solution using the table feature of a graphing utility or a graphical solution using the zoom and trace features of a graphing utility \_\_\_\_\_.

***What you should learn***

How to approximate limits of functions graphically and numerically

**IV. One-Sided Limits** (Pages 829–830)

Describe a **one-sided limit**. The limit at  $c$  of the function  $f(x)$  as  $x$  approaches  $c$  from either just the left or just the right. A limit from the left is denoted as  $\lim_{x \rightarrow c^-} f(x) = L$ . A limit from the

right is denoted as  $\lim_{x \rightarrow c^+} f(x) = L$ .

***What you should learn***

How to evaluate one-sided limits of functions

**Existence of a Limit**

If  $f$  is a function and  $c$  and  $L$  are real numbers, then  $\lim_{x \rightarrow c} f(x) = L$

if and only if \_\_\_\_\_ both the left and right limits exist and are equal to  $L$  \_\_\_\_\_.

**V. Limits from Calculus** (Pages 831–832)

For any  $x$ -value, the limit of a *difference quotient* is an expression of the form

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Direct substitution into the difference quotient always produces \_\_\_\_\_ the indeterminate form  $0/0$  \_\_\_\_\_.

***What you should learn***

How to evaluate limits from calculus

**Homework Assignment**

Page(s)

Exercises