Course Number

Instructor

Date

Section 12.4 Limits at Infinity and Limits of Sequences

Objective: In this lesson you learned how to evaluate limits at infinity and find limits of sequences.

What you should learn
How to evaluate limits of functions at infinity

I. Limits at Infinity and Horizontal Asymptotes
(Pages 845–848)

Define **limits at infinity.**

If f is a function and L_1 and L_2 are real numbers, then the statements $\lim_{x \to -\infty} f(x) = L_1$ and $\lim_{x \to \infty} f(x) = L_2$ denote the limits at infinity.

The first is read "the limit of $f(x)$ as x approaches $-\infty$ is L_1," and the second is read "the limit of $f(x)$ as x approaches ∞ is L_2."

To help evaluate limits at infinity, you can use the following.

If r is a positive real number, then $\lim_{x \to \infty} \frac{1}{x^r} =$ ___0___.

If x^r is defined when $x < 0$, then $\lim_{x \to -\infty} \frac{1}{x^r} =$ ___0___.

Example 1: Find the limit: $\lim_{x \to \infty} \frac{1 + 5x - 3x^3}{x^3}$

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If $f(x)$ is a rational function and the limit of f is taken as x approaches ∞ or $-\infty$,

- When the degree of the numerator is less than the degree of the denominator, the limit is ___0___.
- When the degrees of the numerator and the denominator are equal, the limit is ___the ratio of the coefficients of the highest-powered terms___.
- When the degree of the numerator is greater than the degree of the denominator, the limit ___does not exist___.

II. Limits of Sequences (Pages 849–850)

> ***What you should learn***
> How to find limits of sequences

For a sequence whose nth term is a_n, as n increases without bound, if the terms of the sequence get closer and closer to a particular value L, then the sequence is said to ______converge______ to L. A sequence that does not converge is said to ______diverge______.

Give the definition of the limit of a sequence.

Let f be a function of a real variable, such that $\lim_{x \to \infty} f(x) = L$.

If $\{a_n\}$ is a sequence such that $f(n) = a_n$ for every positive integer n, then $\lim_{n \to \infty} a_n = L$.

Example 2: Find the limit of the sequence $a_n = \frac{(n-3)(4n-1)}{4-3n-n^2}$.

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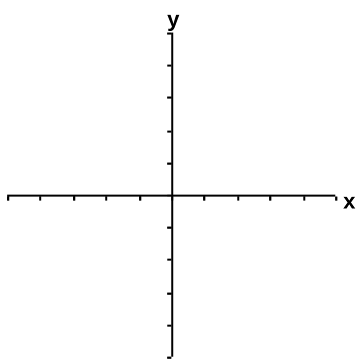

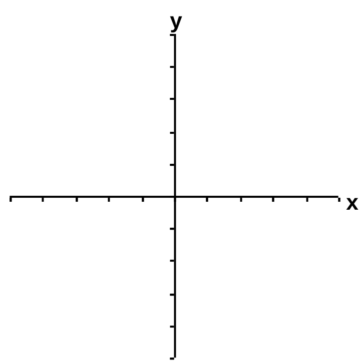

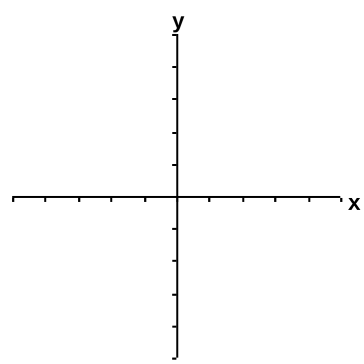

Homework Assignment

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