

Section 12.5 The Area Problem

Objective: In this lesson you learned how to find limits of summations and use them to find areas of plane regions.

Course Number

Instructor

Date

I. Limits of Summations (Pages 854–856)

The following summation formulas and properties are useful for evaluating finite and infinite summations.

What you should learn

How to find limits of summations

$$1. \sum_{i=1}^n c = \underline{cn}, \text{ } c \text{ is a constant.}$$

$$2. \sum_{i=1}^n i = \underline{n(n+1)/2}$$

$$3. \sum_{i=1}^n i^2 = \underline{n(n+1)(2n+1)/6}$$

$$4. \sum_{i=1}^n i^3 = \underline{n^2(n+1)^2/4}$$

$$5. \sum_{i=1}^n (a_i \pm b_i) = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$$

$$6. \sum_{i=1}^n ka_i = k \sum_{i=1}^n a_i, \text{ } k \text{ is a constant.}$$

To find the limit of a summation, begin by applying summation formulas and properties to simplify S . Then use the techniques from the previous section to find the limit as $n \rightarrow \infty$.

Example 1: Find the limit of $S(n)$ as $n \rightarrow \infty$.

$$S(n) = \sum_{i=1}^n \frac{i-5}{n^3}$$

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II. The Area Problem (Pages 857–859)

Describe the area problem.

The area problem is to find the area of the region R bounded by the graph of a nonnegative, continuous function f , the x -axis, and the vertical lines $x = a$ and $x = b$.

What you should learn
 How to use rectangles to approximate areas of plane regions and to use limits of summations to find areas of plane regions

The basic strategy for finding the area of the region R is to use a collection of rectangles of equal width that approximates the region R .

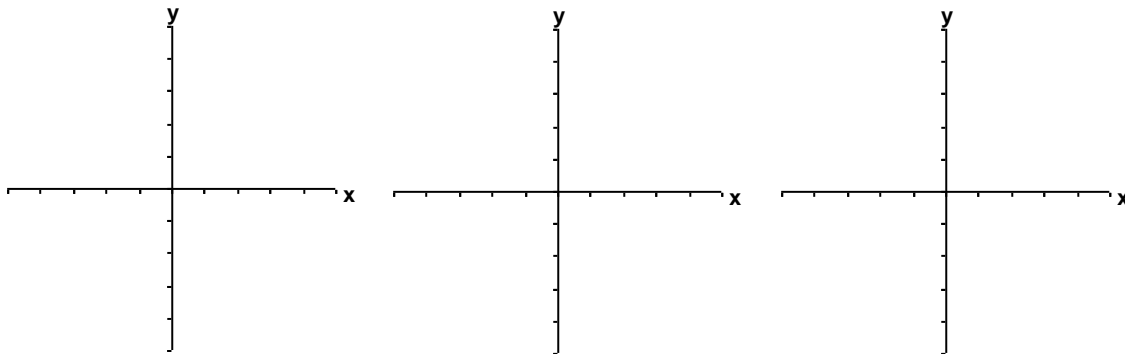
The exact area of a plane region R is the limit of the sum of n rectangles as n approaches ∞ .

Let f be continuous and nonnegative on the interval $[a, b]$. The area A of the region bounded by the graph of f , the x -axis, and the vertical lines $x = a$ and $x = b$ is given by

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(a + (b - a)i / n) [(b - a) / n]$$

Example 2: Find the area of the region bounded by the graph of $f(x) = (x - 4)^2 + 5$ and the x -axis between $x = 3$ and $x = 6$.

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Homework Assignment

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Exercises